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THE FATIGUE OF A SINGLE MENTAL FUNCTION

No subject of experimental investigation has produced such diverse and contradictory results as that of mental fatigue. The first difficulty lies in the impossibility of isolating it from muscular and sensory fatigue. When this is reduced to a minimum, the experimenter finds that he is measuring not only a loss due to fatigue, but a gain due to practice. If this effect is reduced by continued practice there still remains the problem of determining how much of the loss measured is due to fatigue; *i.e.,* inability to work, and how much to boredom, or disinclination to work.

The most reliable results have come from the direct measurement of loss of efficiency in one mental function. Perhaps the purest mental function which can be thus employed is that used to multiply mentally. First used by Thorndike,[1] this method was employed by Miss Arai,[2] in an extended investigation. Miss Arai first practised herself in the mental multiplication of four-figure number by four-figure numbers, with both numbers in view. A test of about eight hours duration produced but little fatigue. Miss Arai next practised the mental multiplication of similar numbers with both numbers held in the memory. After six days of practice, this work was done on four successive days of twelve hours each, pausing only to write down the answer and the time.

The amount of fatigue is shown by the increase in time required to do work of equal accuracy at the end of the day as compared with that required at the beginning of the day. Thorndike estimated Miss Arai's efficiency in the last half hour period of each day to be not less than 75% of her initial efficiency.

The purpose of the experiments outlined below has been to repeat and extend Miss Arai's experiments to ascertain whether this remarkable resistance to mental fatigue is a demonstration of facility in mental multiplication approaching genius or a condition

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[1]Thorndike, E. L., "Mental Fatigue". J.E.P.., 2 (Feb. 1911), pp. 61-80.

[2]Arai, Tsuru, "Mental Fatigue". T. C. Contributions to Education, No. 54. New York. Columbia University. 1912.

to be found in the work of an individual of average abilty in mental multiplication, and to include in the experiments records of changes in pulse rates and body temperature, subjective feelings of boredom, climatic conditions and the amount of fatigue transferred.

The subject T.W.M., is a graduate student in Education at the College of Education, University of Toronto, a grade teacher in a Toronto elementary school, in whose classes there have always been some pupils whose speed and accuracy in mental multiplication exceeded that of their master.

Since Miss Aria's original problems were no longer available, new ones were formed by chance, zeros being omitted. The following examples indicate the range of difficulty:

2212×9481;	5116×8996;
3579×6328;	6839×9847.

The distribution of easy and difficult problems was by chance.

On July 6, 1930 the repetition was begun, following the course of Miss Arai's experiment as closely as was possible. The subject first practised the mental multiplication of numbers like those above with both numbers in sight. As soon as the answer was obtained it was written down and the time recorded by stop-watch and clock time. Then without pause the next problem was begun.

The system of multiplication used in all the experiments was chosen because it seemed the easiest, the ability of the subject to do the problems being very much in doubt. It was as follows:

Example: 4782 x 3956

3000 x 4000	12,000,000	
3000 x 700	2,100,000	14,100,000
3000 x 80	240,000	14,340,000
3000 x 2	6,000	14,346,000
900 x 4000	3,600,000	17,946,000
900 x 700	630,000	18,576,000
900 x 80	72,000	18,648,000
900 x 2	1,800	18,649,800
50 x 4000	200,000	18,849,800
50 x 700	35,000	18,884,800
50 x 80	4,000	18,888,800
50 x 2	100	18,888,900
6 x 4000	24,000	18,912,900
6 x 700	4,200	18,917,100
6 x 80	480	18,917,580
6 x 2	12	18,917,592

The average time per problem on the first day was 6.5 minutes and the average error, 2.5 per problem. On the fourth day these were 3.8 and 2.5 respectively. Thus practised, the subject tried to produce fatigue by multiplying sixty such problems in the same manner with a rest for dinner after the 52nd problem. No fatigue measureable by the problem, time or error units was produced.

The work was then made more difficult. Instead of having the numbers to be multiplied in view, the subject looked at them only long enough to read them once, and multiplied them from memory. If they were forgotten, they were looked at again and the problem begun again from the beginning, the time for the futile attempt being included in the time required to do the problem. After seven practice periods, there followed four twelve-hour periods, on July 18, 19, 20 and 21, 1930, from about 10.00 a.m. to about 10.00 p.m., without any rest except the few moments required to write down the answer and to record the time. On the first day 108 problems were done; on the second, 110; on the third, 111; and on the fourth, 111.

Failing to produce extreme mental fatigue by this method, the experiment was extended in July of the next year. New problems, slightly more difficult were formed, this time omitting the ones as well as the zeros. All the possible combinations of the numbers 2 to 9 were used, without any figure recurring in any combination. The following are examples of this type of problem:

9684×4739; 5946×2869;
7925×7846; 4368×2657.

Sufficient material was prepared for ten days of continuous mental work in periods of twelve hours per day.

No practice periods were included. On July 12, 1931, the subject began the series with the intention of continuing as long as it was found possible and considered safe to do so. On the first day 104 such problems were done, showing a loss due to lack of practice over a period of about one year, and in some measure due to the increased difficulty of the problems. The loss was recovered on the second day when 120 problems were done. The peak of output was reached on the third day when 123 problems were done. From that day on the number of problems done decreased, the output for the fourth to eigth days being 114, 105, 100, 86 and 69, respectively. On the seventh day the subject began to show physical reactions.

These became so marked on the eighth that it was decided to conclude the experiment.

Before and after each work period of all the experiments described above and in addition, 31 periods of general mental work in July and August, 1930, the subject recorded the pulse rate, the body temperature, and the climatic conditions. Multiplication tests of an easier type, of three-figures by three-figures, multiplied with the numbers in view, were made before and after the work-periods. Tests were also made to ascertain if the fatigue induced by the work of a special mental function were transferred to another special function. These tests consisted of the memorization of foreign equivalents of English words. German words were used in the 1930 tests and Spanish in the 1931 tests. Four lists of ten words each were used before and after each work period.

The results of these experiments are presented here only in the form of summary tables. Full details are to be found in the copies of the complete dissertation on file in the library of the Ontario College of Education and the main library of the University of Toronto.

TABLE I

T.W.M.'s time for mental multiplication of four-figure numbers by four-figure numbers with both numbers held in the memory, in twelve-hour periods of continuous work, July 18, 19, 20, 21, 1930. Time is in minutes required for each successive group of four problems with errors included in the time by Thorndike's weighting formula.[1] Where the last group contains fewer than four problems, the time given is that which would have been required to complete the group at the same rate and with the same degree of accuracy as was maintained in that part of the group which was completed.

Problems	July 18/30	July 19/30	July 20/30	July 21/30	Mean
1- 4	23.4	17.1	13.0	21.1	18.65
5- 8	19.8	20.0	14.8	30.4	21.25
9- 12	23.2	21.6	19.6	25.1	22.375
13- 16	19.2	21.7	16.0	18.6	18.875
17- 20	25.8	28.8	16.1	27.3	24.5
21- 24	24.8	20.8	15.7	31.6	23.225
25- 28	25.5	26.5	17.9	22.8	23.175
29- 32	21.8	21.6	29.7	16.9	22.5
33- 36	29.4	24.1	22.2	24.9	25.15
37- 40	22.1	28.6	27.3	22.4	25.1
41- 44	23.9	24.9	22.6	23.5	23.75
45- 48	19.5	18.4	20.5	25.3	20.925
49- 52	22.1	24.2	32.9	21.5	25.175
53- 56	22.5	19.3	24.5	21.8	22.25
57- 60	32.7	16.6	22.0	19.4	22.675
61- 64	24.7	29.7	24.6	25.3	26.75
65- 68	28.7	27.0	25.0	31.9	28.15
69- 72	20.7	23.5	20.4	27.4	25.5
73- 76	28.5	21.6	36.5	31.7	29.575
77- 80	27.3	18.1	31.4	25.7	25.625
81- 84	26.8	22.5	29.5	25.5	26.75
85- 88	25.0	21.3	25.5	23.0	23.7
89- 92	20.0	21.3	24.3	28.1	23.425
93- 96	38.0	27.2	30.1	28.2	30.875
97-100	38.2	36.9	31.8	28.0	33.725
101-104	27.0	33.6	39.6	37.0	34.3
105-108	27.1	32.8	25.5	43.0	32.1
109 and 110 in proportion		45.4			
109, 110 and 111 in proportion			31.7	30.4	35.833
First 4	23.4	17.1	13.0	21.1	18.65
Last 4	27.1	43.7	32.0	31.2	33.5
First 8	43.2	37.1	27.8	51.5	39.9
Last 8	54.1	76.8	58.8	79.8	67.375
Mean	25.8	24.5	24.7	26.3	

[1]Thorndike's weighting formula: To the total time for the four problems add 3% of the total time for each error more than two in any answer, and for each error less than two in any answer subtract 3% of the total time.

TABLE II

T.W.M.'s time for mental multiplication of four-figure numbers by four-figure numbers, with both numbers held in the memory, in twelve-hour periods of continuous work, July 12, 13, 14, 15, 16, 17, 18, 19, 1931. Time is in minutes required for each successive group of four problems with errors included in the time by Thorndike's weighting formula (see footnote of Table I, p. 7). Where the last group contains fewer than four problems, the time given is the weighted time for the number of problems completed.

Problems	July 12	July 13	July 14	July 15	July 16	July 17	July 18	July 19	Mean
1- 4...	27.8	22.6	17.7	16.2	20.2	19.0	11.9	17.1	19.06
5- 8...	26.9	20.1	14.8	24.1	15.5	28.3	28.3	31.3	23.66
9- 12...	27.3	21.0	19.0	14.9	15.6	24.9	26.4	31.3	22.55
13- 16...	26.1	16.7	17.8	20.7	18.2	37.0	32.5	37.1	25.76
17- 20...	19.8	18.2	14.9	19.8	24.1	19.5	21.9	26.2	20.55
21- 24...	22.9	15.8	15.8	25.9	21.3	21.3	23.1	34.1	22.52
25- 28...	15.7	19.1	21.6	17.4	23.9	23.9	23.2	47.3	24.01
29- 32...	24.7	24.1	16.5	17.3	21.8	18.2	29.2	39.4	23.90
33- 36...	23.6	14.3	17.5	25.5	23.1	22.2	20.9	37.6	23.09
37- 40...	24.7	24.4	25.9	17.8	24.5	30.5	30.3	68.2	30.79
41- 44...	28.0	19.1	21.0	18.6	24.9	21.2	27.3	38.1	24.77
45- 48...	21.7	24.2	19.1	20.7	26.5	23.6	35.4	31.7	25.36
49- 52...	21.4	25.0	24.1	14.5	22.3	26.4	31.6	56.4	27.71
53- 56...	22.1	20.7	20.0	26.3	28.9	21.8	27.6	43.9	26.41
57- 60...	25.9	20.9	25.9	20.9	24.3	34.7	27.2	37.5	27.16
61- 64...	22.6	19.8	23.2	33.1	21.9	27.2	33.0	32.9	26.71
65- 68...	31.9	23.2	15.1	33.7	23.7	29.0	30.0	48.6	29.40
69- 72...	22.8	30.9	13.7	30.7	25.8	18.4	25.9	14.3	24.03
73- 76...	42.8	21.1	22.0	24.5	25.7	25.1	38.9 (69 only)		28.58
77- 80...	31.4	25.3	18.0	22.9	26.9	25.9	51.3		28.81
81- 84...	22.4	34.4	26.2	25.3	39.2	23.8	51.5		31.83
85- 88...	20.7	31.4	19.5	30.2	28.7	34.9	20.6		27.57
89- 92...	27.7	27.1	20.1	25.8	33.6	30.1	85 and 86		27.40
93- 96...	28.5	26.8	23.9	29.5	25.6	32.9			27.87
97-100...	22.6	19.4	18.9	33.5	35.5	34.6			27.42
101-104...	28.6	27.2	21.8	24.4	20.7				24.54
105-108...		25.7	21.7	24.2	8.9				23.86
109-112...		20.5	23.0	24.0	(105 only)				22.50
113-116...		23.9	33.0	14.9					28.45
117-120...		20.8	19.0	(113 and 114)					19.90
First 4....	27.8	22.6	17.7	16.2	20.2	19.0	11.9	17.1	19.06
Last 4....	28.6	20.8	27.6	28.3	23.9	34.6	39.9	54.1	32.23
First 8....	54.7	42.7	32.5	40.3	35.7	47.3	40.2	48.4	42.73
Last 8....	51.2	44.7	46.8	52.4	59.1	67.5	102.4	87.3	63.93
Mean.....	25.4	24.12	20.84	23.8	27.4	26.2	30.1	39.0	27.69

TABLE III

Coefficients of fatigue obtained by comparing equal amounts of work done at the beginning and at the end of each twelve-hour period, with the degree of accuracy and the time required combined in one measure, using the formula

Coefficient of fatigue $= \frac{T^2 - T^1 \times 100}{T^1}$ where T^1 is the time required at the beginning of the period, and T^2 the time required at the end.

	—July 18 to 21, 1930—				—July 12 to 19, 1931—							
Units of work compared	1st day	2nd day	3rd day	4th day	1st day	2nd day	3rd day	4th day	5th day	6th day	7th day	8th day
First 4 and last 4 problems...	15.8	155.5	146.1	47.9	2.9	−8.0	56.0	74.7	18.3	82.1	235.3	216.4
First 8 and last 8 problems...	25.2	107.0	111.5	54.9	−6.4	4.7	44.0	30.0	15.3	42.7	154.7	80.4
First hr. and last hr. of work.	30.6	58.1	70.0	45.6	−2.5	6.2	30.4	30.0	50.9	37.3	115.9	85.0
Time for memorizing 40 foreign equivalents of English words before and after work	5.0	19.7	4.8	31.0	20.4	26.4	50.7	12.4	45.1	34.0	51.1	17.6

TABLE IV

Boredom and Efficiency

Correlations between the subjective feelings of fatigue and the objective measurement of efficiency. (Pearson r)

Between the average degree of subjective fatigue felt over the twelve-hour period and the average number of problems done correctly per hour	−.7128
Between the increase in the subjective feeling of fatigue in the twelve-hour period and the increase in time required to multiply each three-figure problem before and after the work-period	+.064
Between the increase in the subjective feeling of fatigue in the twelve-hour period and the increase in time required to memorize each foreign equivalent before and after the work-period	+.369

TABLE V

Practice Effect

Average net gain in per cent. determined by comparing the weighted time required for the first four problems of each work-period with that of the first four problems of the preceding period.

		Average net gain in per cent.
Exper. I, Part I.	5 periods, both numbers in view	34.575%
Exper. I, Part II.	7 practice periods, both numbers held in the memory	9.466%
	4 test periods, both numbers held in the memory	.075%
Extension Exper.	8 test periods, both numbers held in the memory:	
	First three days	15.666%
	Last five days	−5.100%
Percentage of practice loss between the conclusion of Exper. I, Part II, July 21, 1930, and the first day of the Extension Exper., July 12, 1931, with no intervening practice periods.		−31.750%

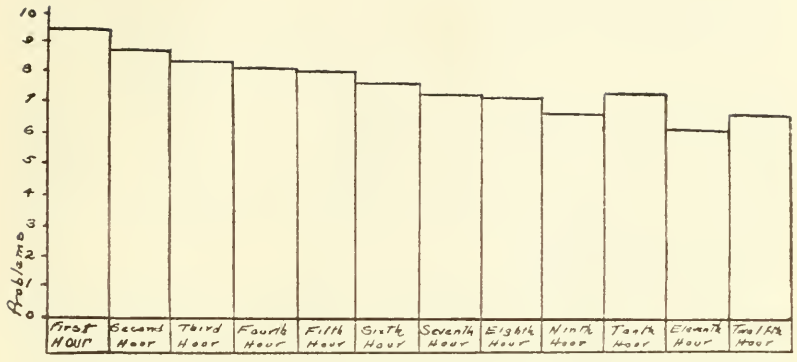

DIAGRAM 1

THE ONSET OF FATIGUE

The curve of work according to the average number of problems done correctly during each successive hour of the twelve twelve-hour work periods of continuous mental multiplication of four-figure numbers by four-figure numbers with both numbers held in the memory.

CONCLUSIONS

1. While decreased pulse rates are almost invariably recorded at the conclusion of long periods of mental work, the decreases can be attributed to the physical inactivity accompanying mental work.

2. Changes in body temperature were too slight and too variable to permit any reliable conclusion.

3. Changes in temperature, relative humidity and hours of sunlight produced no measureable effect upon mental efficiency.

4. In the case of the subject T. W. M., when the primary function fatigued is that used to multiply mentally and the secondary function tested for transferred fatigue is that used to memorize foreign equivalents of English words, the amount of fatigue transferred to the secondary function is roughly 30% of that produced in the primary function.

5. In the case of the subject T. W. M., when the function used to multiply mentally is exercised for eighty-four hours and twenty minutes, in 16 periods of varied length, in doing 764 problems, four figures by four figures, the amount of practice gain retained after one year without practice, is 68%. Compared with this the degree of permanence of the memory of German or Spanish equivalents is less than 10%.

6. While the curve of work for any one period is quite erratic, the mean curves show the onset of fatigue to be gradual. When the work is continued for a long period of time the onset of fatigue is accelerated in the last half of the period.

7. Mental incompetency seems almost impossible to achieve. Even when four-figure by four-figure multiplication is continued, mentally, with the figures retained in the memory, for twelve-hours daily without rest, for eight consecutive days, the efficiency of the subject at the end of the day is never less than 50% of the initial efficiency. The small amount of fatigue produced by this difficult and disagreeable task is in some measure due to the experimental conditions and to the determination of the subject to maintain effort at an even level to obtain true results. Thus by sheer will power it seems that hidden sources of energy may be drawn upon and utilized as a kind of emergency ration which seemingly prevents

the onset of fatigue, or at least, offsets the effects of boredom for a considerable time.

8. The feelings of boredom, or subjective fatigue, appear to be fair, though unreliable measures of the state of efficiency. When the work period is long and the work difficult, these feelings invariably exaggerate the amount of fatigue. The last problem done on the eighth and last day of the 1931 series, 5429 × 7284, was multiplied mentally from memory in 14.7 minutes, with one error. The last four problems of that day were done in a weighted time of 32.23 minutes. Yet at the time the subject felt that every moment the work continued was a torture. What we casually term "fatigue" seems to be mostly boredom.

9. While real fatigue is very difficult to produce it does not follow that it is wise to drive mental efficiency to the limit of endurance. It has yet to be proved that a mental function, once completely fatigued, can recover its efficiency as does a muscular function. Although it is no doubt true that many persons give up hard tasks far too soon, and fail to achieve results that are within their grasp, it would be wise to consider boredom to be a natural automatic safeguard against the fatigue of a mental function beyond a point where its recovery is doubtful.

"Let not a man force a habit on himself with a perpetual continuance, but with some intermission: for both the pause reinforceth the new onset; and if a man that is not perfect be ever in practice, he shall as well practise his errors as his abilities, and induce one habit of both;—In Studies, whatsoever a man commandeth upon himself, let him set hours for it; but whatsoever is agreeable to his nature, let him take no care of any set times; for his thoughts will fly to it of themselves—"[1]

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